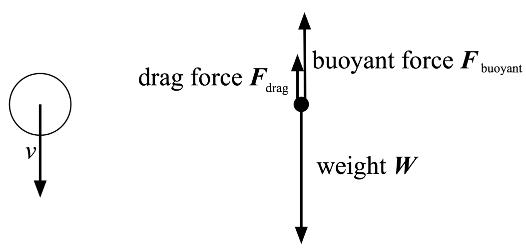
Terminal velocity of a falling bead

Effective weight

A bead of radius R is falling through a fluid under the influence of gravity. We want to establish a relationship between the bead's characteristics (its size and density), the fluid's characteristics (its density and viscosity) and the terminal velocity of the bead.

To begin with, we draw down the forces acting on the bead:



Here the weight and buoyant force are given by

$$\begin{aligned} W &= m_{\text{bead}} g = \rho_{\text{bead}} V g \\ F_{\text{buoyant}} &= m_{\text{fluid}} g = \rho_{\text{fluid}} V g, \end{aligned}$$

where the ρ's are densities (in kg/m^3), V is the bead's volume, and g is the acceleration due to gravity (9.81 m/s^2). You may not have seen the buoyant force written down this way, but we all know that when you submerge something in water its effective weight goes down; if the object is denser than water it will still sink (net force downward) but if it is less dense than water it will float (net force upward). This emerges naturally from the effective weight of the submerged bead:

$$W_{\text{effective}} = W - F_{\text{buoyant}} = (\rho_{\text{bead}} - \rho_{\text{fluid}}) V g.$$

Drag force

There are two commonly used expressions for the drag force F_{drag}:

$$\begin{aligned} F_{\text{viscous}} &= -\left(6\pi\mu R v\right)\hat{v} \\ F_{\text{aerodynamic}} &= -\left(\tfrac{1}{2} C_d \rho_{\text{fluid}} A v^2\right)\hat{v}, \end{aligned}$$

where μ is the viscosity of the fluid (in N.s/m^2), R is the radius of the spherical bead (in m), $\vec{v}$ is the velocity of the bead (in m/s, so that v is the speed and $\hat{v}$ is the unit vector pointing in direction of velocity), C_d is the drag coefficient (a unitless number that depends on the shape of the object but is almost constant; for a sphere at the speeds we will see in lab, $0.4 < C_d < 1$), and A is the cross-sectional area of the bead (in m^2). We are trying to determine which of these two expressions for F_{drag} most accurately describes the behavior of the beads in our experimental situation.

Terminal velocity

Since our beads are denser than water, if a bead starts from rest it will accelerate downwards. As it picks up speed, the drag force (which opposes motion, and is therefore upward) will increase until it completely cancels the effective weight. From that time onward, the net force on the bead is zero so the bead's velocity becomes constant; this is called the *terminal velocity.* Mathematically, terminal velocity is defined as the velocity at which

$$W_{\text{effective}} = F_{\text{drag}}.$$

Keep in mind that the drag force depends on the velocity v.

Reynolds number

The *Reynolds number* Re is a quantitative measure of whether objects moving in a fluid are "fast and large" (Re $\gg 1$) or "slow and small" (Re $\ll 1$). Technically Re is the ratio of inertial to viscous forces, but for our purposes it is simply a number that divides motion into the "fast" and "slow" regimes. For an object of size a, the Reynolds number is defined as

$$\text{Re} \equiv \frac{\rho_{\text{fluid}} v a}{\mu}$$

For a sphere, the "size" a is equal to the radius.

Physical properties

For reference, the various physics parameters we will need are

density of water	ρ_{water}	1000 kg/m^3
density of glycerine	$\rho_{\text{glycerine}}$	1260 kg/m^3
density of steel	ρ_{steel}	8050 kg/m^3
density of aluminum	ρ_{aluminum}	2700 kg/m^3
density of nylon	ρ_{nylon}	1150 kg/m^3
viscosity of water	μ_{water}	8.90×10^{-4} N.s/m^2
viscosity of glycerine	μ_{water}	1.4 N.s/m^2